Exam 3 Outline

Calculus of Real-Valued Functions of Several Variables

I. Functions of Several Variables

II. Limits and Continuity

III. Partial Derivatives

IV. Differentiability and Tangent Planes

V. The Gradient

   1. Directional Derivatives and the Gradient
   2. Geometric Interpretation of the Gradient

VI. The Chain Rule

VII. Optimization in Several Variables

   1. Critical Points
   2. The Second Derivative as a Matrix
   3. Second Derivative Test via Eigenvalues

VIII. Lagrange Multipliers

IX. Taylor's Series for Functions of Several Variables

Exam 3 Formula Sheet

- For a regular curve $\vec{r}(t)$,
  - $\vec{T}(t) = \dfrac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}$
  - $\vec{N}(t) = \dfrac{\vec{T}\,'(t)}{|\vec{T}\,'(t)|}$
  - $\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$
- For a regular curve $\vec{r}(t)$, the curvature is given by

$$\kappa(t) = \frac{|\vec{T}\,'(t)|}{|\vec{r}\,'(t)|} = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}.$$

- **The Second Derivative Test for Functions of Two Variables**
  Left $f(x, y)$ be a twice differentiable function, and assume its second partial derivatives are continuous. Let (a, b) be a critical point for f, and define the Hessian of f at (a, b) to be

$$D(a, b) = f_{xx}(a, b) f_{yy}(a, b) - [f_{xy}(a, b)]^2 .$$

  - If $D(a, b) < 0$, then (a, b) is a Saddle Point.
  - If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then (a, b) is a Local Minimum.
  - If $D(a, b) > 0$ and $f_{xx}(a, b) < 0$, then (a, b) is a Local Maximum.
  - If $D(a, b) = 0$, the test is inconclusive.
- **Lagrange Multipliers**
  If a differentiable function f has a local maximum or local minimum on a constraint curve of the form $g =$constant (where g is a differentiable function), then there is a constant λ so that

$$\nabla f = \lambda \nabla g$$

  at the local extremum (so long as ∇g is not zero at the point in question).